Pafnutii Lvovich CHEBYSHEV (or TCHÉBICHEF)
b. 16 May 1821 (o.s.) - d. 26 November 1894 (o.s.)

Summary Chebyshev is regarded as the founder of the St. Petersburg School of mathematics, which encompassed path-breaking work in probability theory. The Chebyshev Inequality carries his name; he intitiated rigorous work on a general version of the Central Limit Theorem.

Chebyshev was born in the village of Okatovo, Borovskii uezd (district of the town of Borovsk), Kaluga ġubernia. His education prior to the move of the family to Moscow in 1832 was at the home of his parents in Okatovo. His mother taught him reading and writing amongst other things; but arithmetic and the French language were taught to him by a female cousin, Avdotiia Kvintillianovna Sukhareva described (Prudnikov, 1976, pp. 18 and 30) as an educated girl and a governess to the Chebyshev children. It is instructive to note now when women's roles are more clearly appreciated, that the history of mathematics may owe much to this woman; Chebyshev kept a picture of her all his life. Nikoforovsky (1992, p. 130) notes that, indeed, Chebyshev completely mastered French, and said that he thought out his works in French, and only then composed them in Russian. In considering scientific problems, he had the habit of lapsing into French. This explains much of his subsequent francophilia.

After enrolling at Moscow University in 1837, Chebyshev came under the mathematical influence of Nikolai Efimovich Zernov (1804-1862) and Nikolai Dmitrievich Brashman (1796-1866). The inspiration of Brashman on Chebyshev devoting himself to mathematics and Chebyshev's veneration of him, is well-known (he kept a photograph which Brashman had given him until his own death). From Zernov Chebyshev learned "pure mathematics"; according to Nikoforovsky (1992, p. 126) "$\cdots$ a solid knowledge of the foundations of mathematics, enabling him to conduct independent research". He could hardly have had a better-qualified teacher: Zernov was the first in Russia to obtain the degree of Doctor of Mathematical Sciences (in 1837).

The year 1841 was notable in that Chebyshev completed his university course, and began to prepare for his magisterial exams. On 17 June (o.s.) 1841, at a commemorative ceremony at Moscow University, Brashman read an address entitled "On the influence of the mathematical sciences on the development of intellectual capabilities" which was in part a stimulus for both the probabilistic works: Buniakovsky's book "Foundations of the Mathe-

matical Theory of Probabilities" of 1846 (the first probability treatise in the Russian language), and Chebyshev's magisterial thesis, defended in 1846, but apparently published in 1845: "An Essay on Elementary Analysis of the Theory of Probabilities."

This thesis (for a reprinting see Chebyshev, 1955) gives as its motivation, dated 17 October (o.s.) 1844:

> "To show without using transcendental analysis the fundamental theorems of the calculus of probabilities and their main applications, to serve as a support for all branches of knowledge, based on observations and evidence - that was the topic put to me for realization by his eminence the lord guardian Count Sergei Grigorievich Stroganov ⋯"

Sergei Grigorievich Stroganov (1794-1882) was not a supervisor in the normal technical sense. An aristocrat and capable general, in the tradition of the times he was appointed to a sequence of top administrative roles. In particular, in the period 1835-1847 was guardian [popechitel'] of the Moscow Educational Region. According to the *Entsyklopedicheskii Slovar:* "Contemporaries describe these years as a golden era for Moscow University, since he personally became involved in raising the quality of its professoriate, and had a gift for finding talented people whom he then supported and protected . . . . . ."

We can see that Chebyshev was fortunate in coming to Moscow University as student in 1837, and that he was one of those favoured and encouraged by Stroganov, possibly through the intervention of Brashman.

In the event the magisterial dissertation was entirely theoretical. Its impact at the time appears not to have been large. It does give, as promised, an elementary but rigorous analytical discussion of the then principal aspects of probability theory, requiring no calculus, only algebra; and directs the reader to the use of a supplied table of values for applications. It led to a remarkable paper published in 1846 giving an analytical deduction of the Weak Law of Large Numbers of Poisson, which, in spite of the eminence of the journal (*Crelle's J. Reine Angew. Mathematik*), passed unnoticed among the French mathematicians to whom the Law remained an object of controversy. Both the dissertation and paper display a new feature within probability theory, the estimation of deviation from the limit.

The young Chebyshev, unable to find a suitable teaching job in Moscow after his magisterial dissertation moved to St. Petersburg, where he started

lecturing on higher algebra and number theory in September, 1847. Buniakovsky seems to have taken him under his wing soon after, and they worked together on a new edition of Euler's *Theory of Numbers*. Buniakovsky lectured on probability theory at
St. Petersburg University from 1850 to his retirement in 1859; when the course was taken over by Chebyshev. He remained a firm supporter of Chebyshev until his own death, proposing him for the Academy of Sciences, and acting as an intermediary in St. Petersburg for correspondence between Chebyshev and western scientists since Chebyshev was a notoriously bad correspondent. Buniakovsky's own francophilia is well-known; from the beginning of his mathematical creativity he wrote papers in French, and the noted economist and philosopher P.B. Struve in 1918 called him "a Russian disciple of the French mathematical school". This would have accorded well with, and indeed influenced, Chebyshev, strengthening the background provided by his cousin Sukhareva, and the general French cultural influences in the Russian Empire of the time. Buniakovsky also had a strong interest in the theory of mechanisms, a topic on which Chebyshev was to become famous.

The decade from 1850 (when he was appointed extraordinary professor) to 1860 was one of intensive activity for Chebyshev. In June-November 1852 he was on study leave in France, Belgium, Germany and England. From our point of view (statistics and probability) the most important people he met were Cauchy and Bienaymé in Paris.

In a context of interpolation which can be regarded as *statistical* theory, all 3 names became linked through an important paper of Chebyshev of 1859, apparently originally in French, on *linear least squares* and *orthogonal polynomials*. This followed publication in 1858 of a translation by Bienaymé into French of a paper in Russian of 1855 by Chebyshev, with a view to its use as ammunition for Bienaymé's controversy on interpolation with Cauchy. This translation may have been stimulated by the personal contact between Chebyshev and Bienaymé in 1852, and again in 1856. The French influence in general, and that of Bienaymé in particular on Chebyshev's *probabilistic* work in St. Petersburg was facilitated by Chebyshev's friend Nikolai Vladimirovich Khanykov [1819-1878].

The inequality commonly known as Chebyshev's Inequality, and less commonly as the Bienaymé-Chebyshev Inequality, was published by Chebyshev simultaneouly in Russian in St. Petersburg, and (in French translation by Khanykov) in Paris in Liouville's journal (*Journal de Mathématiques Pures*

et Appliquées **12**, 177-184), in 1867.

Bienaymé's proof was published in 1853 in the *C.R.Acad.Sci., Paris* **37**, 5-13, within his now best known paper: "Considérations à l'appui de la découverte de Laplace sur la loi de probabilité dans la méthode des moindres carres.". It is reprinted in Liouville's journal immediately preceding (pp. 158-176) the French version of Chebyshev's paper. There is a small editorial comment by Liouville which does not relate to the Inequality. However, the implication of the juxtaposition of the papers is clear.

Chebyshev spent quite some time after 1867 searching for the essence of Bienaymé's 1853 proof of the Bienaymé-Chebyshev Inequality, finally formulating in 1874 the "method of Bienaymé". Indeed, this method is essentially what later came to be called the "Method of Moments". Chebyshev later used this method to prove the first version of the Central Limit Theorem for sums of independent but not identically distributed summands in 1887, his final and great achievement in probability theory, which was then quickly taken up by his students Markov (q.v.) and, in another
direction, Liapunov, thus making the Russian Empire a focus of probability theory for the world.

More generally, Chebyshev is credited with founding the great "Petersburg School" of mathematics, with which are associated (in addition to Markov and Liapunov) the names of A.N. Korkin, E.I. Zolotarev, K.A. Posse, D.A. Gravè, G.F. Voronoi, A.V. Vasiliev, V.A. Steklov, and A.N. Krylov, amongst others; and there were many
"descendants" of the School in the next generation, such as M.P. Kravchuk (Krawtchouk) (q.v.).

It is interesting to note that Chebyshev's eminence was late in being recognised. Although his death is noted in a modest anonymous entry in *Nature* (August 8, 1895, p.345), he is not mentioned in Vol.XXII (1905) of the *Russkii Biograficheskii Slovar*, even though lesser mathematicians and other Chebyshevs are.

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E. Seneta